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A study of cumulative fatigue damage in AISI 4130 steel

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Experimental data were obtained using AISI 4130 steel under stress ratios of −1 and 0. A study of cumulative fatigue damage using Miner's and Kramer's equations for stress ratios of −1 and 0 for low-high, low-high-mixed, high-low, and high-low-mixed stress sequences has revealed that there is a close agreement between the theoretical and experimental values of fatigue damage and fatigue life. Kramer's equation predicts less conservative and more realistic cumulative fatigue damage than the popularly used Miner's rule does.

1. Introduction

Failure of structural components in service is invariably due to fatigue, creep and stress corrosion. Fatigue failure is the consequence of repeated or fluctuating loads varying over a wide range exerted on a part or a system. Such failure invariably starts at the surface in the form of a crack and propagates to the core of the component until sudden rupture occurs. When the stress amplitude is constant and the variation is cyclic, the life of the component can be determined using the standard *S–N* (applied stress against fatigue life) diagram available in the literature. Such determination is impossible when the stress amplitude is not constant (block loading).

Miner and Palmgren [1] were the first to propose the cumulative fatigue damage rule known as Miner's Rule for the prediction of failure of a component subjected to stresses of varying amplitude over a given set of cyclic blocks. If σ_1, σ_2, σ_3 . . . represent the stress, amplitudes applied to a part, and n_1, n_2, and n_3 . . . represent the corresponding number of cycles, Miner's Rule may be stated as

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} \ldots = 1 \qquad (1)$$

where N_1, N_2, N_3 . . . are the number of cycles to failure obtained from the *S–N* diagram at stresses σ_1, σ_2, σ_3 . . . , respectively. Various other theories [2–8] have been proposed but none of these can accurately predict fatigue damage for most of the commonly encountered loading situations. Miner's theory, which is the most commonly used, does not take into account the previous history of the material in multilevel loading.

It has been suggested that when a material is fatigued, the damage is confined in the surface layer in the form of work hardening [9–13]. As the number of cycles or the stress amplitude increases, there is an increase in the surface layer stress. As the fatigue damage accumulates, the surface layer stress attains critical value and a crack is formed independent of the stress amplitude, leading to fatigue failure. Kramer [9] has suggested that cumulative fatigue damage can be expressed in terms of the rate of increase in the surface layer stress with the number of cycles. Since the critical surface stress is constant for a given material, it is only necessary to determine the contribution to the surface stress due to fatigue at a given stress for a given number of cycles and to sum up such contributions. Therefore

$$\sum \frac{\sigma_s}{\sigma_s^*} = D \qquad (2)$$

where σ_s and σ_s^* are the surface layer stress and critical surface layer stress, respectively, and D is the cumulative damage. The failure will occur when D equals unity. Kramer further extended his investigation [10] and proposed the failure equation in the following form

$$\frac{n_1\sigma_1^p}{\beta} + \frac{n_2\sigma_2^p}{\beta}\left(\frac{\sigma_1}{\sigma_2}\right)^{pf_1} + \frac{n_3\sigma_3^p}{\beta}\left(\frac{\sigma_2}{\sigma_3}\right)^{pf_1}\left(\frac{\sigma_1}{\sigma_2}\right)^{pf_1f_2} + \ldots = 1 \qquad (3)$$

where $p = -1/m$, m is the slope of the *S–N* diagram which is of the form $\sigma = CN^m$, and $\beta = C^p$. p and β are material constants and f represents damage histories in the previous stress sequences.

$$f_1 = \frac{\sigma_1^p n_1}{\beta}$$

$$f_2 = \frac{\sigma_2^p n_2}{\beta}\left(\frac{\sigma_1}{\sigma_2}\right)^{pf_1}$$

Details of the derivation of the equation may be found in [14, 15].

The purpose of the present investigation was to obtain fatigue data for AISI 4130 steel and to compare the experimental values of cumulative fatigue damage and fatigue life with those predicted using Miner's and Kramer's equations.

2. Experimental work

AISI 4130 steel was used in this investigation. This material is extensively used in the aerospace industry because of its superior properties such as high strength to weight ratio, dimensional stability, etc. Tables I and

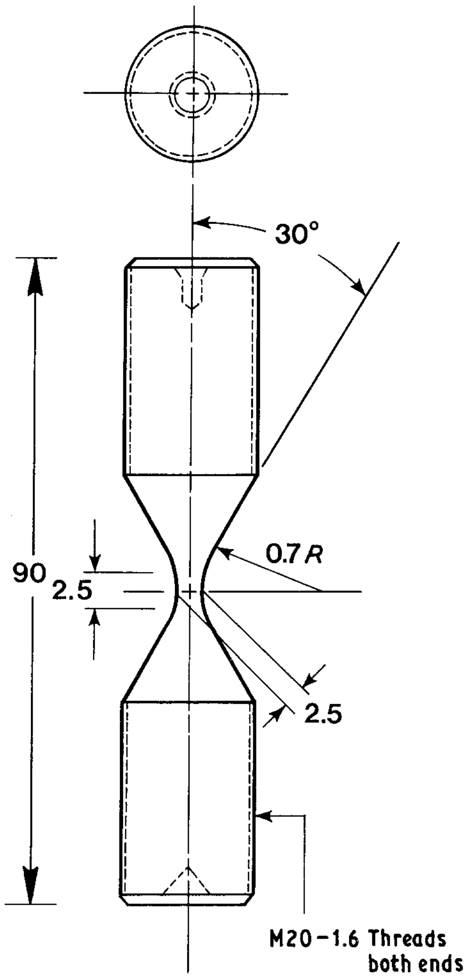

Figure 1 Fatigue specimen. (All dimensions are in mm.).

II show the chemical composition and mechanical properties of the alloy lot used. The design of the fatigue specimen is shown in Fig. 1. The specimens were machined from extruded solid bar of 0.75 in (20 mm) diameter. The gauge section was formed on Tensilkut and Tensilgrind. The Tensilkut was used to take a rough cut with a two flute milling cutter, while the Tensilgrind was used to finish grinding the specimen with a 120 grit silicon carbide wheel. The specimens were then batch heat treated under the following conditions:

solutionizing at	1575° F (857° C) for 2 h;
quenching,	water quench;
ageing,	1000° F (537° C) for 3 h;
cooling,	air cool.

The gauge section of the specimen was then mechanically polished with 180–1200 grit silicon carbide papers to reduce the tool marks and other surface irregularities. This was followed by electropolishing to obtain an even surface finish using the following conditions:

TABLE I chemical composition of AISI 4130 steel (wt %)

Fe	C	Mn	P	S	Si	Cr	Mo
Balance	0.305	0.500	0.035	0.04	0.275	0.950	0.20

electrolyte,	70% ethanol (absolute), 12% distilled water, 10% 2-butoxy ethanol, 8% perchloric acid (60%);
bath temperature,	− 30° C;
stirrer speed,	4–5 r.p.m.;
voltage,	30–65 V;
time,	1 min.

After electropolishing, the specimens were examined under a microscope for circumferential scratches and other stress raisers.

Fatigue tests were conducted on a direct tension compression machine which is equipped with an automatic hydraulic load maintainer. The data for the *S–N* diagram was generated for the stress ratios of − 1 and 0 where

$$\text{stress ratio} = R = \frac{\text{minimum stress}}{\text{maximum stress}}$$

The testing of the specimens to generate cumulative fatigue data was done in four stages. In the first, second and third stages the loads and the number of cycles applied at each load were predetermined. In the fourth stage the load was preselected and the test was continued until the specimen failed. Tests were conducted with low-to-high, low-to-high-mixed, high-to-low, and high-to-low mixed stress sequences. All the test were conducted in the elastic range of the material.

3. Discussion of results

A considerable amount of data has been generated concerning the cumulative fatigue damage study in AISI 4130 steel at various stress ratios. In the following, only selected data representative of the results in general are presented.

The fatigue strength against fatigue life (*S–N*) curves plotted for stress ratios of − 1 and 0 are shown in Fig. 2. The values of the constants for each *S–N* curve are also shown in Fig. 2.

Table III shows the cumulative of a fatigue data for completely reversed stress condition $R = -1$, for the low-high-low-high-mixed, high-low, and high-low-mixed stress sequences. The number of cycles shown against each stress level is the average of the values of at least five identical tests. For specimen 1, the stress was 465 MPa in the first stage for 200 000 cycles, 605 MPa in the second stage for 10 000 cycles, 730 MPa for 1000 cycles in the third stage, and finally, stressed

TABLE II Mechanical properties of AISI 4130 steel

Ultimate tensile strength (MPa)	Yield strength (MPa)	Shear strength (MPa)	Poisson's ratio	Modulus of elasticity in tension (MPa)	Modulus of elasticity in compression (MPa)	Modulus of elasticity in torsion (MPa)
1232	1036	756	0.32	203 000	203 000	77 000

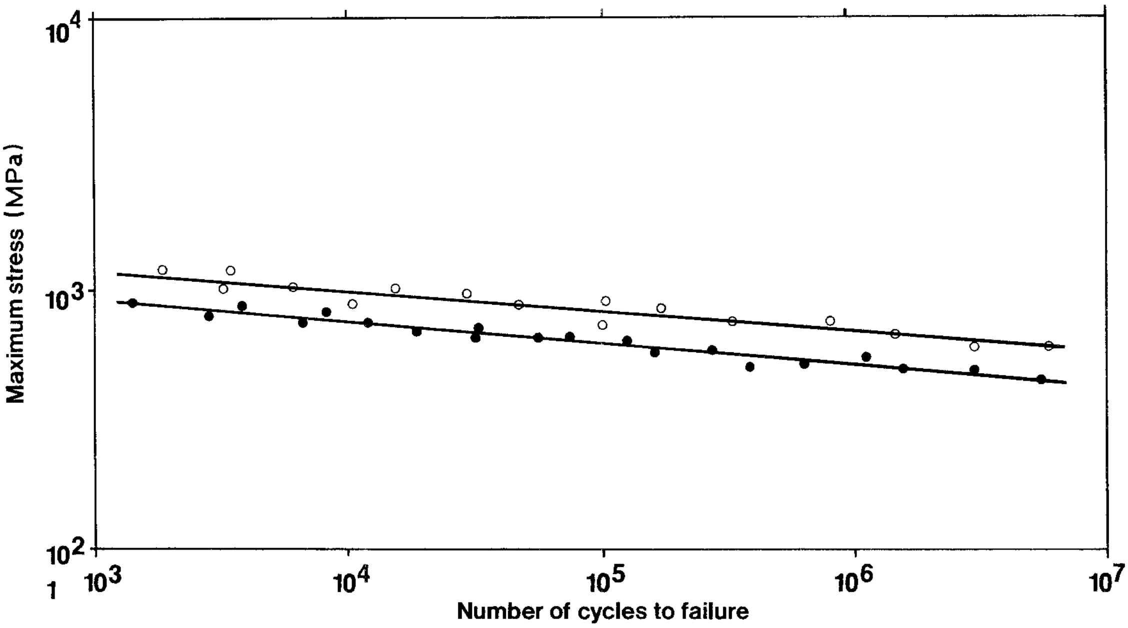

Figure 2 *Sø–N* curves for AISI 4130 steel. (○): $R = -1$, $m = -9.45638e/02$, $c = 235610$, $p = 10.57486$, $\beta = 6.459E + 56$; (●): $R = 0$, $M = -9.573046e/-02$, $c = 313479$, $p = 10.446$, $\beta = 2.590559E + 57$.

TABLE III Cumulative fatigue data for a stress ratio of -1 (Material, AISI 4130 Steel)

Specimen number	Cumulative stress									Stress sequences†
	Stage 1		Stage 2		Stage 3		Stage 4*			
	Stress (MPa)	Cycles	Stress (MPa)	Cycles	Stress (MPa)	Cycles	Stress (MPa)	Number of cycles: Experimental	Number of cycles: Kramer	
1	465	200 000	605	10 000	730	1000	800	1200	1142	L-H
2	455	200 000	595	10 000	700	2000	875	600	616	L-H
3	490	100 000	630	10 000	770	1000	840	600	591	L-H
4	455	200 000	700	5000	595	800	770	1700	1808	L-H-M
5	490	100 000	630	10 000	560	1000	700	4800	4735	L-H-M
6	420	200 000	595	5000	490	3000	700	23 900	24 279	L-H-M
7	840	500	700	1500	560	5000	630	45 390	40 389	H-L
8	770	500	630	6000	560	30 000	420	398 200	349 626	H-L
9	700	1000	630	10 000	560	10 000	490	62 900	63 659	H-L
10	700	500	490	50 000	545	30 000	420	12 890	127 996	H-L-M
11	770	900	630	2000	700	5000	560	800	925	H-L-M
12	630	10 000	455	100 000	560	10 000	420	160 300	150 228	H-L-M

*Specimen stressed until failure.
†L-H = low-high; L-H-M = low-high-mixed; H-L = high-low; H-L-M = high-low-mixed.

TABLE IV Cumulative fatigue damage for stress ratio of -1 (Material, AISI 4130 Steel)

Specimen	D_t, Kramer*	D_t, Miner	N_F (expermimental)†	N_F (theory)‡	N_F (exp) / N_F (theory)	Stress sequence§
1	1.0226	1.3006	212 200	212 142	1.0002	L-H
2	0.9878	1.4195	212 600	212 615	0.99	L-H
3	1.0054	1.4427	111 600	111 591	1.00	L-H
4	0.9674	1.3795	207 500	207 607	0.99	L-H-M
5	1.0073	1.2143	115 800	115 739	1.0005	L-H-M
6	0.9885	1.1974	258 900	259 279	0.9985	L-H-M
7	0.9944	0.6481	47 389	52 389	0.9045	H-L-M
8	0.9987	0.6489	384 700	386 126	0.9963	H-L-M
9	0.9969	0.7739	83 900	84 659	0.991	H-L
10	1.0022	0.8821	209 400	208 497	1.0043	H-L-M
11	0.9953	0.9501	8700	8824	0.9859	H-L-M
12	1.0063	0.6953	280 300	270 228	1.0372	H-L-M

*$D_t = f_1 + f_2 + f_3 + f_4$ is the total cumulative fatigue damage.
†N_F (experimental) is the total number of cycles to failure.
‡(theory) is the total number of cycles to failure using Kramer's equation.
§L-H = low-high; L-H-M = low-high-mixed; H-L = high-low; H-L-M = high-low-mixed.

TABLE V Cumulative fatigue data for a stress ratio of 0 (Material, AISI 4130 Steel)

Specimen number	Cumulative stress									
	Stage 1		Stage 2		Stage 3		Stage 4*			
	Stress (MPa)	Cycles	Stress (MPa)	Cycles	Stress (MPa)	Cycles	Stress (MPa)	Number of cycles		Stress sequences†
								Experimental	Kramer	
1	465	467 200	605	189 200	730	104 500	800	6100	5981	L-H
2	455	635 800	595	232 300	700	5500	750	11 000	10 839	L-H
3	420	813 400	490	467 200	630	189 200	770	70 100	169 127	L-H
4	455	610 000	700	29 500	595	32 400	770	46 800	47 579	L-H-M
5	490	467 200	630	189 200	560	289 000	770	82 400	83 053	L-H-M
6	420	813 500	595	232 300	490	467 200	700	360 200	355 689	L-H-M
7	840	4800	700	45 600	560	149 000	630	138 500	155 806	H-L
8	770	6100	630	189 200	560	467 200	420	520 200	595 198	H-L
9	700	32 500	630	189 200	560	89 000	490	682 300	698 146	H-L
10	700	49 500	490	667 200	595	332 300	420	272 400	275 444	H-L-M
11	770	1900	630	108 200	700	29 500	560	647 300	650 173	H-L-M
12	630	239 200	445	635 800	560	359 000	420	1800	2281	H-L-M

*Specimen stressed until failure.
†L-H = low-high; L-H-M = low-high-mixed; H-L = high-low; H-L-M = high-low-mixed.

at 800 MPa until failure in the fourth stage. The number of cycles predicted using Kramer's equation for failure to occur is also shown in the fourth stage. Specimens 1–3 were tested with low-high sequences specimens 4–6 were tested with low-high-mixed sequences, specimens 7–9 were tested with high-low mixed stress sequences. It can be seen that there is a close agreement between the experimental and predicted values of fatigue life for the specimens tested under all the stress sequences used in this investigation.

Table IV shows the cumulative fatigue lives determined using Kramer's as well as Miner's equation for the specimens reported in Table III. The table also shows the total number of cycles to failure obtained experimentally and theoretically using Kramer's equation. It can be seen that for all the stress sequences used in this work, Kramer's equation predicts cumulative fatigue damage closer to unity than the Miner's rule does.

Table V shows the cumulative fatigue data generated using a stress ratio of 0.00, i.e. tension only. The stress sequences used in this investigation are shown in the extreme right column of the table. The experimental value of fatigue life agrees well with that obtained using Kramer's equation for all the tests reported in this table.

Table VI shows the cumulative fatigue damage and total fatigue life obtained experimentally and predicted using Kramer's equation. Once again it can be seen that Kramer's equation predicts cumulative damage closer to unity than the Miner's equation and that there is a close agreement between the total number of fatigue cycles obtained experimentally and predicted using Kramer's equation.

4. Conclusions

The following conclusions are drawn from the results of this investigation.

1. For the stress ratios of −1 and −0, there is a close agreement between the fatigue lives determined experimentally and predicted using the Kramer's equation for the low-high, low-high-mixed, high-low, and high-low-mixed stress sequences.
2. The value of cumulative fatigue damage predicted using Kramer's equation is in good agreement with that obtained experimentally for all the stress sequences used in this investigation.

TABLE VI Cumulative fatigue damage for stress ratio of 0 (Material, AISI 4130 Steel)

Specimen	D_t, Kramer*	D_t, Miner	N_F (expermimental)†	N_F (theory)‡	N_F (exp) / N_F (theory)	Stress sequence§
1	1.0012	1.5579	767 000	766 881	1.000	L-H
2	1.0101	1.1030	879 650	879 489	1.000	L-H
3	1.0097	1.7557	1 539 900	1 638 927	0.939	L-H
4	0.9877	1.1070	718 718	719 497	0.999	L-H-M
5	0.9974	1.2107	1 027 800	1 028 453	0.999	L-H-M
6	1.0067	1.1639	1 873 200	1 868 684	1.002	L-H-M
7	0.9997	0.6095	337 900	355 206	0.951	H-L
8	0.9975	0.6110	1 182 700	1 257 698	0.940	H-L
9	0.9960	0.7901	993 000	1 008 846	0.984	H-L
10	0.9990	0.8373	1 321 400	1 324 444	0.998	H-L-M
11	0.9970	0.8755	786 900	789 773	0.996	H-L-M
12	0.9900	0.7967	1 235 800	1 236 281	1.000	H-L-M

*$D_t = f_1 + f_2 + f_3 + f_4$ is the total cumulative fatigue damage.
†N_F (experimental) is the total number of cycles to failure.
‡(theory) is the total number of cycles to failure using Kramer's equation.
§L-H = low-high; L-H-M = low-high-mixed; H-L = high-low; H-L-M = high-low-mixed.

3. Kramer's equation prdicts less conservative and more realistic cumulative fatigue damage than the popularly used Miner's rule does in most cases.

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